Electron-Phonon Interaction Effects on Spectrum of Topological Insulator Thin Films

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Abstract

We study the electron-phonon interaction effects in topological insulator thin films. We find that the band structure of topological insulator thin films is significantly affected by such interaction. In particular, the interplay of the energy band gap induced by the hybridization between the top and bottom surfaces of topological insulator thin films and the one induced by the electron-phonon interaction play important role in generating the effective band structure of the system. Moreover, we investigate the effect of temperature on the band structure.

Introduction:

Topological insulators (TIs) are an emerging class of quantum matter that has attracted a considerable attention in condensed matter physics and materials science. They possess gapped states in the bulk and gapless non-trivial boundary conducting states[1,2], including edge states of 2DTIs and surface states of 3DTIs. Such states are originated from intrinsic properties such as spin-orbit coupling, band inversion, bulk bands of opposite parities and are topologically protected by time-reversal symmetry. These conducting states in TIs are robust against disorder/impurities scattering and many-body interactions, leading to their potential applications in spintronics [3-6].

At low energies, these gapless surface states play very important role in the transport processes which are described by the massless Dirac equation in relativistic quantum mechanics. TI thin films have attracted significant attention because of their potential applications [7–9]. In this work, we study the electron-phonon interaction effects on the spectrum of topological insulator thin films.

Model Hamiltonian:

The Hamiltonian for our system

$$H = H_o + H_{ph} + H_{e-ph}$$

$$H_o = \sum_K [v_F(\sigma_x p_y - \sigma_y p_x) + \Delta\sigma_Z] C_k C_k^{\dagger}$$

$$H_{ph} = \hbar\omega_q \sum_q a_q^{\dagger} a_q$$

$$H_{e-ph} = \sum_{kq} M_q C_k C_k^{\dagger}(a_q + a_q^{\dagger})$$

Eigen values:

$$E_\lambda = \lambda\sqrt{\hbar^2 k^2 v_F^2 + \Delta^2}$$

Eigen states:

$$\psi_{\lambda,k}(r) = \frac{e^{i\mathbf{k}.\mathbf{r}}}{\sqrt{2s}}\begin{pmatrix}\sqrt{1+\lambda cos\theta} \\ -ie^{i\varphi k}\lambda\sqrt{1-\lambda cos\theta}\end{pmatrix}$$

Strength of coupling :

$$[M(q)]^2 = \frac{M_0^2}{q \in (q)} e^{-2qd}$$

Where

$$M_0^2 = \pi e^2 \omega_{SO}\left[\frac{1}{\epsilon_\infty+1} - \frac{1}{\epsilon_o-1}\right]$$

Density of states:

$$D_o(\omega) = \frac{2\omega_{SO}}{\omega^2-\omega_{SO}^2}$$

And

$$F_{kk'}^{\lambda\lambda'} = |F_{\lambda k}^{\dagger} F_{\lambda' k+q}|^2 = \frac{1}{2}[1+\lambda\lambda'[cos^2\theta\,(1-cos\varphi_{kk'}) - cos\varphi_{kk'}]]$$

$$\sum_\lambda(k,\omega) = -\frac{1}{\beta}\sum_{\lambda'}\sum_{q,iv_n}\left[M_{kq}^{\lambda\lambda"}\right]^2 D(q,i\omega_n)G_{0,\lambda'}(k+q, i\omega_n+iv_n).$$

with

$$\beta = \frac{1}{k_B T},\; G_{0,\lambda}(k,i\omega_n) = \frac{1}{i\omega_n-\xi_{k\lambda}} \equiv \text{bare Green's function}$$

$$M_{kq}^{\lambda\lambda"} \equiv \text{the interaction matrix element}$$

After the standard analytical continuation from $i\omega_n$ to $\omega + i0^+$ the retarded self energy is given by

$$\text{Re}[\textstyle\sum_\lambda(k,\omega)] = -\pi M_0^2 \sum_{\lambda'}\int \frac{k'dk'}{(2\pi)^2} G_{\lambda\lambda'}(k,k')\left[\frac{N_o+n_F(\xi_{k'\lambda'})}{\omega+\omega_{so}-\xi_{k'\lambda'}} + \frac{N_o+1-n_F(\xi_{k'\lambda'})}{\omega-\omega_{so}-\xi_{k'\lambda'}}\right]$$

$$N_o = \frac{1}{e^{\beta\omega_{SO}-1}},\; n_F(k_\lambda) = \frac{1}{e^{\beta\xi_{k\lambda}+1}},$$

$\xi_{k\lambda} = E_\lambda - \mu$, with μ as the chemical potential

The effective spectrum of TIs thin films assisted by phonon is

$$E = E_\lambda + Re[\textstyle\sum_\lambda(k,\omega)]$$

References:

[1] J.E. Moore, Nature **464**, 194 (2010) .
[2] D. Kong, Y. Cui, Nat. Chem. **3**, 845 (2011).
[3]S. Raghu, S.B. Chung, X.-L. Qi, S.-C. Zhang, Phys. Rev. Lett. **104,** 116401 (2010) .
[4] I. Garate, M. Franz, Phys. Rev. Lett. **104,** 146802 (2010) .
[5] I. Garate, M. Franz, Phys. Rev. B **81**, 172408 (2010) .
[6] S. Mondal, D. Sen, K. Sengupta, R. Shankar, Phys. Rev. Lett. **104,** 046403 (2010) .
[7] M. König et al., Science **318**, 766 (2007).
[8] S. Bauer and C. A. Bobisch, Nat. Commun. **7**, 11381 (2016).
[9] C. Nayak, S. H. Simon, A. Stern, M. Freedman and S. Das Sarma Rev. Mod. Phys. **80**, 1083 (2008).